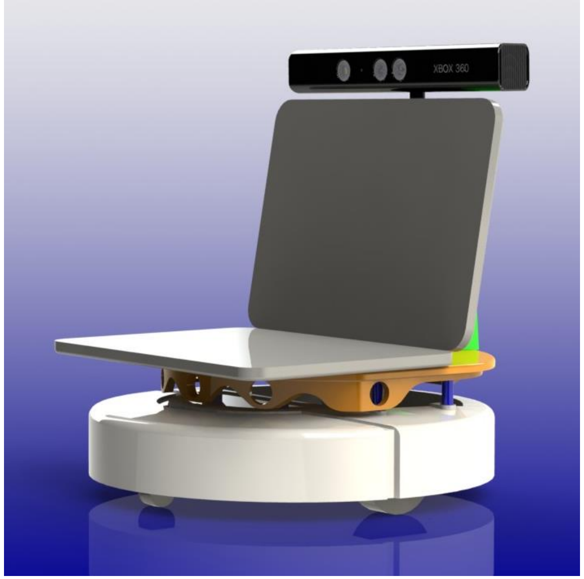
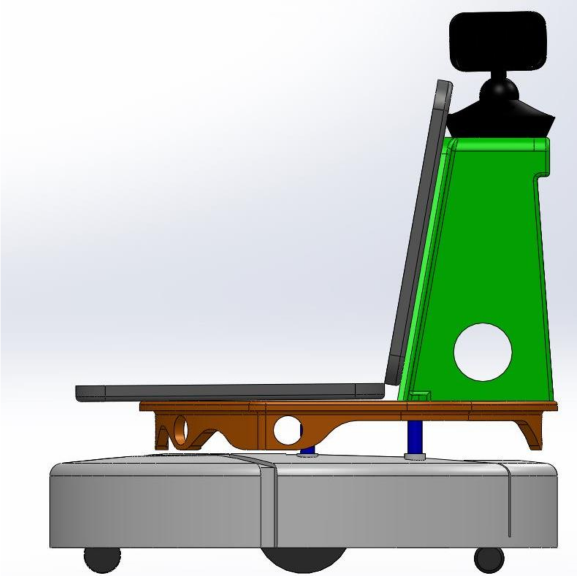
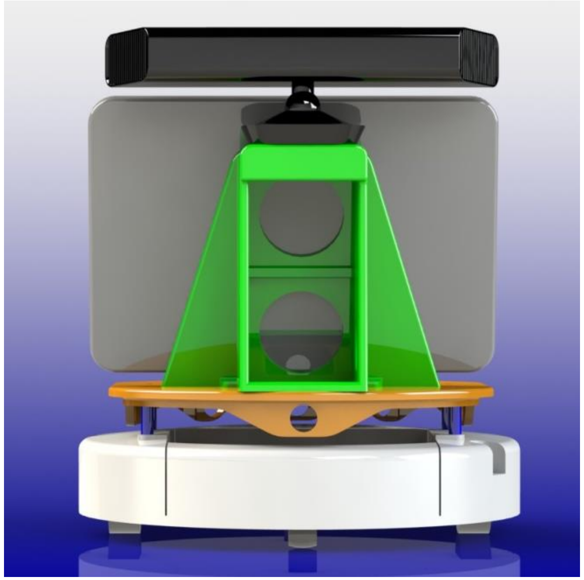
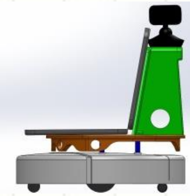
MAE 493G, CpE 493M, Mobile Robotics

5. Measurement and Calibration

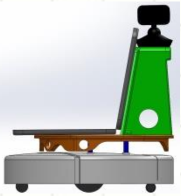




Instructor: Yu Gu, Fall 2013



A Perfect Sensor

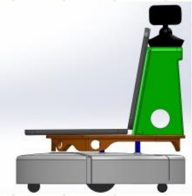
- Connection of the sensor does not distort the measured object itself (high input impedance);
- Output instantly reaches and stabilizes at the measured value (fast response);
- Output is sufficiently large (high sensitivity);
- Device is not sensitive to other parameters (low cross-sensitivity);
- Measures large and small signals (high dynamic range);
- Measures fast and slow changing signals (high bandwidth);
- Output remains at the measured value unless the measured signal itself changes (low noise and drift);
- Output varies in proportion to the signal level of the measured object (static linearity);
- Low size, weight, cost, and power consumption.



Making Good Measurements

- But sensors are never perfect (although many latest digital sensors are quite good);
- A measurement system is also not just about sensors. It includes the power system, signal conditioner, data acquisition hardware, communication device, **operator**, among others;
- Where to place the sensors is also of great importance;

Everything has to work in harmony to get the best results!



Measurement Error

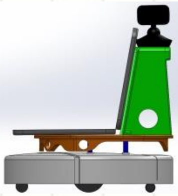
Measurement = Truth + Error

We don’t typically know the Truth…

Error always exist, which includes:

- **Systematic Errors:** Systematic errors occur when there is a problem in the measurement system that affects all measurements in the same way;

- **Random Errors:** Random errors occur because of random and inherently unpredictable events in the measurement process.



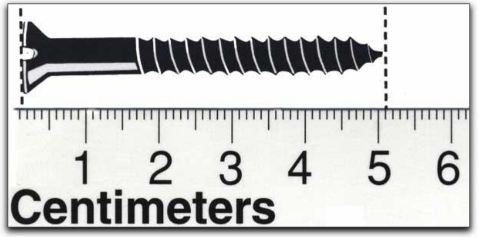
Random Error

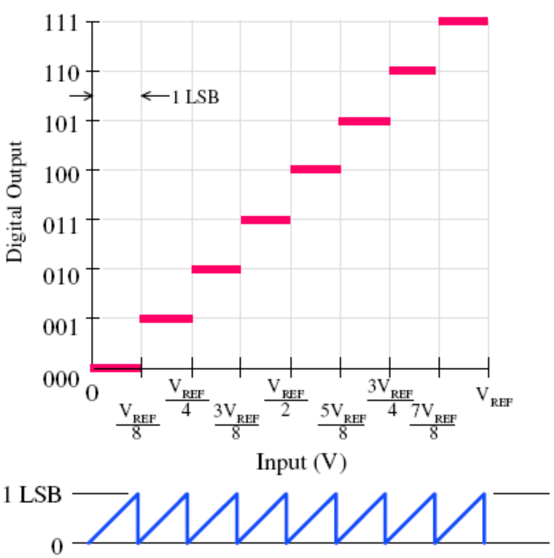
- Caused by any factors that randomly affect measurement of the variable across the samples;
- Each person’s skill, experience, attitude, or mood can affect their performance;
- Random error does not have consistent effects across the entire sample. If we could see all the random errors in a distribution, the mean would be **zero**;
- The important property of random error is that it adds **variability** to the data but does **not affect average** performance for the group.
- Random error can not be removed through calibration.

Random Errors can be Reduced Using Statistical Methods

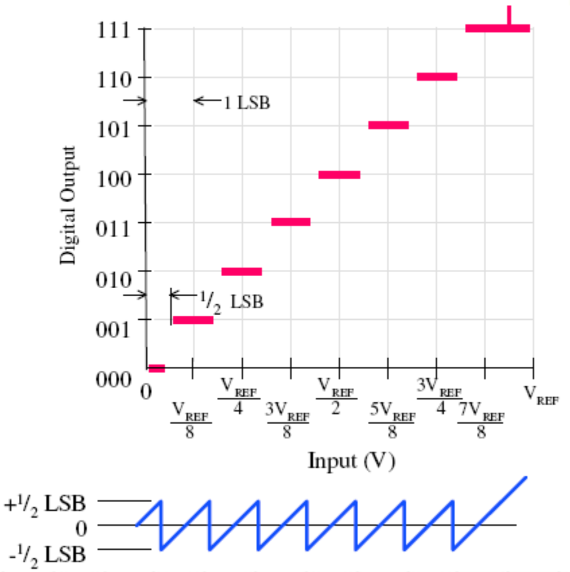
Quantization Error

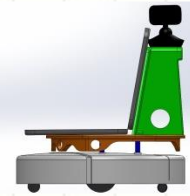
- <u>Quantization error</u> or <u>round-off error</u> is the difference between the actual analog value and quantized digital value;
- The maximum error we have here is 1 LSB (Least Significant Bit). This 0 to 1 LSB range is known as the "<u>quantization uncertainty</u>";
- An error of 0 to 1 LSB is not as desirable as is an error of ±1/2 LSB, so we could introduce a ½ LSB offset into the ADC to force an error range of ±1/2 LSB (change the '*floor*' function to '*round*');
- Quantization error is a form of random error.





The Magnitude of the Error Ranges from Zero to 1 LSB

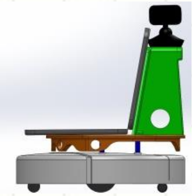




Systematic Error

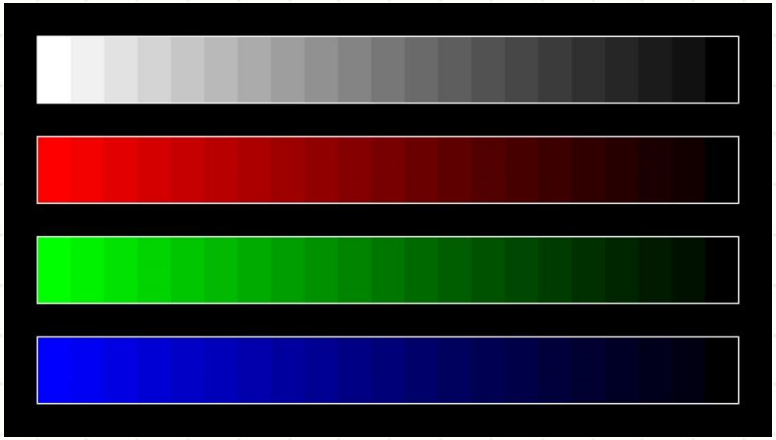
- Systematic error is caused by any factors that systematically affect measurement of the variable across the sample;
- Unlike random error, systematic errors tend to be consistently either positive or negative - because of this, systematic error is sometimes considered to be bias in measurement;
- Systematic errors often occur reproducibly from faulty calibration of equipment or observer bias;
- Statistical analysis in generally is not useful for reducing the systematic errors, but rather corrections must be made based on experimental conditions.

Systematic Errors can be Reduced Through a Careful Experiment Design and System Calibration



Calibration

- Calibration consists of comparing the output of the instrument under test against the output of an instrument of known accuracy when the same input is applied to both instruments;
- This procedure is carried out for a range of inputs covering the whole measurement range of the instrument.
- Calibration ensures that the measuring accuracy of all instruments used in a measurement system is known over the whole measurement range, providing that the calibrated instruments are used in environmental conditions that are the same as those under which they were calibrated.
- For use of instruments under different environmental conditions, appropriate correction has to be made.

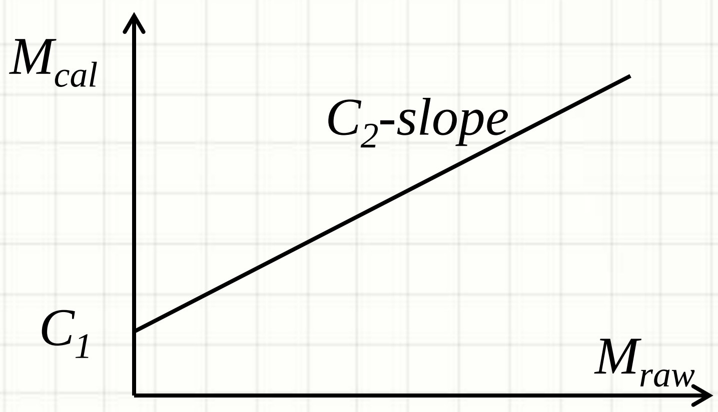


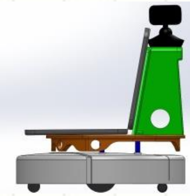
Simple Linear Calibration

- Assuming the input-output relationship of an instrument can be approximated with a linear function, two constants, C_1 and C_2, can be used for calibrating this instrument. C_1 is used as the zero offset value, and C_2 is used as the slope or gain adjustment.
- The calibrated instrument measurement M_{cal} can be determined from the raw measurement M_{raw} with the following relationship:

$$M_{cal} = C_1 + (C_2 \times M_{raw})$$

- How to calculate the parameters C_1 and C_2 is discussed next.





Two Point Calibration Method

- Allow the measurement instrument to warm up and stabilize for an adequate amount of time;
- Use the instrument to measure an accurately known and stable reference value M_{ref1} and record the measurement M_{raw1};
- Repeat Step 2 for another known reference value M_{ref2} and the corresponding recorded measurement M_{raw2}. The two known reference values should be near the bounds of the intended operating range;
- Determine the value of C_1 and C_2 from the following calculations:

$$C_2 = (M_{ref1} - M_{ref2}) / (M_{raw1} - M_{raw2})$$

$$C_1 = M_{ref1} - (M_{raw1} \times C_2)$$

$$M_{cal} = C_1 + (C_2 \times M_{raw})$$

Multi-Point Calibration w/ MATLAB

- Use multiple points for calibration can provide a better understanding for the sensor performance;
- Matlab ‘polyfit’ function is ideal for multi-point calibration.
- A sample MATLAB code is provided below:

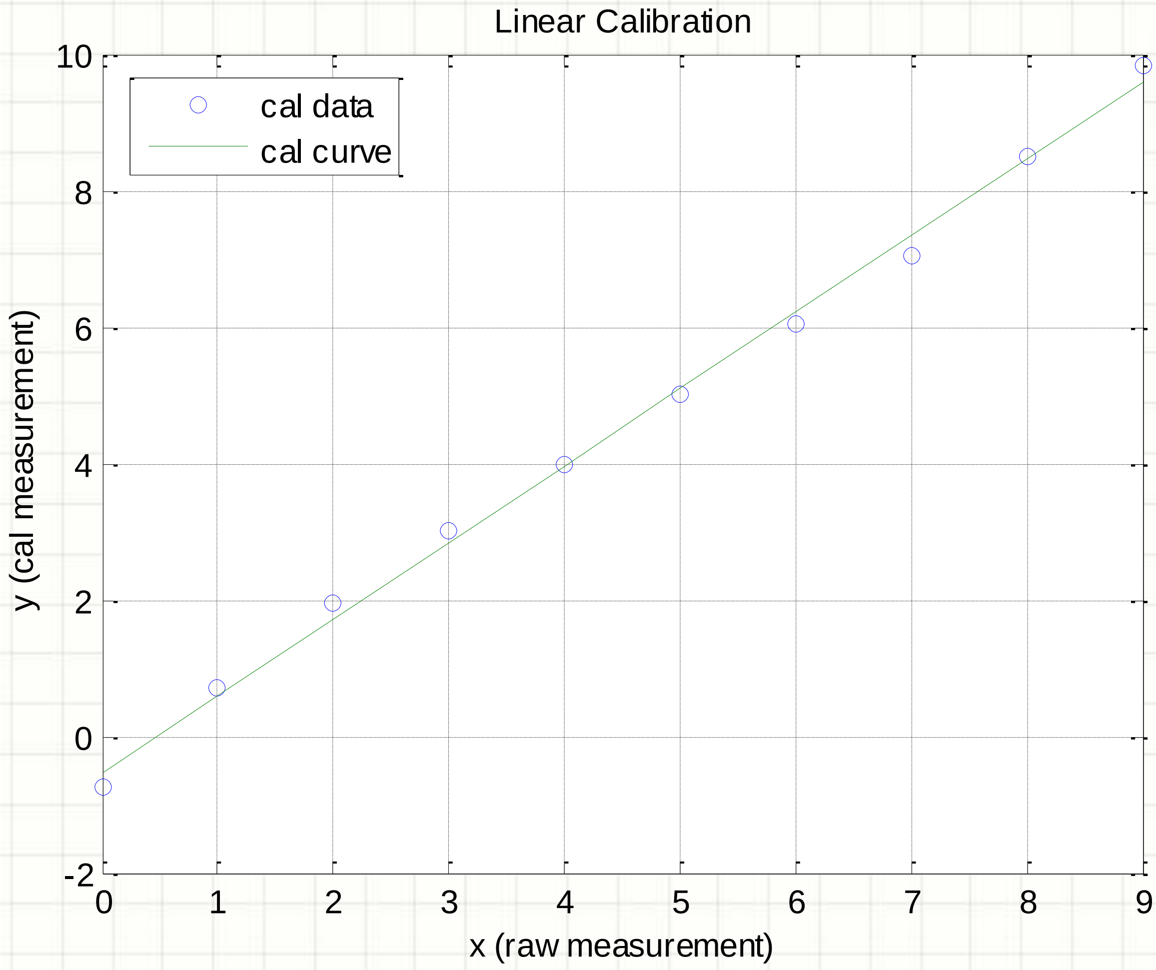
```
x=[0 1 2 3 4 5 6 7 8 9]'; % x is the raw measurement
% y is the measurement from the calibration device
y=[-0.75 0.7 1.95 3.02 3.97 5.01 6.05 7.04 8.5 9.82]';
p = polyfit(x,y,1)  % perform linear calibration
f1 = polyval(p,x);  % evaluate the calibration polynomial
plot(x,y,'o',x,f1,'-')
xlabel('x (raw measurement)');
ylabel('y (cal measurement)');
title('Linear Calibration')
legend('cal data', 'cal curve')
grid on
```

- The results are: p=[1.1231   -0.5229], which means:

$$M_{cal} = -0.5229 + (1.1231 \times M_{raw})$$

Linear Calibration Result

- What if the normal operation range is [2, 7]?



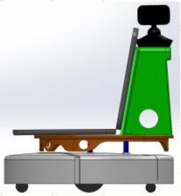
Nonlinear Calibration w/ MATLAB

- Sometime the sensor output is highly nonlinear so the linear calibration may not be adequate.
- Again, the Matlab 'polyfit' function is very useful:

```
x=[0 1 2 3 4 5 6 7 8 9]'; % x is the raw measurement
% y is the measurement from the calibration device
y=[-0.35 0.9 3.25 7.5 15 26 33 45 62 85]';
p1 = polyfit(x,y,1) % perform linear calibration
f1 = polyval(p1,x);
p2 = polyfit(x,y,2) % perform 2nd order calibration
f2 = polyval(p2,x);
plot(x,y,'o',x,f1,'-',x,f2,'r')
xlabel('x (raw measurement)');
ylabel('y (cal measurement)');
title('Linear and 2nd Order Calibration')
legend('cal data', 'linear cal','2nd order')
grid on
```

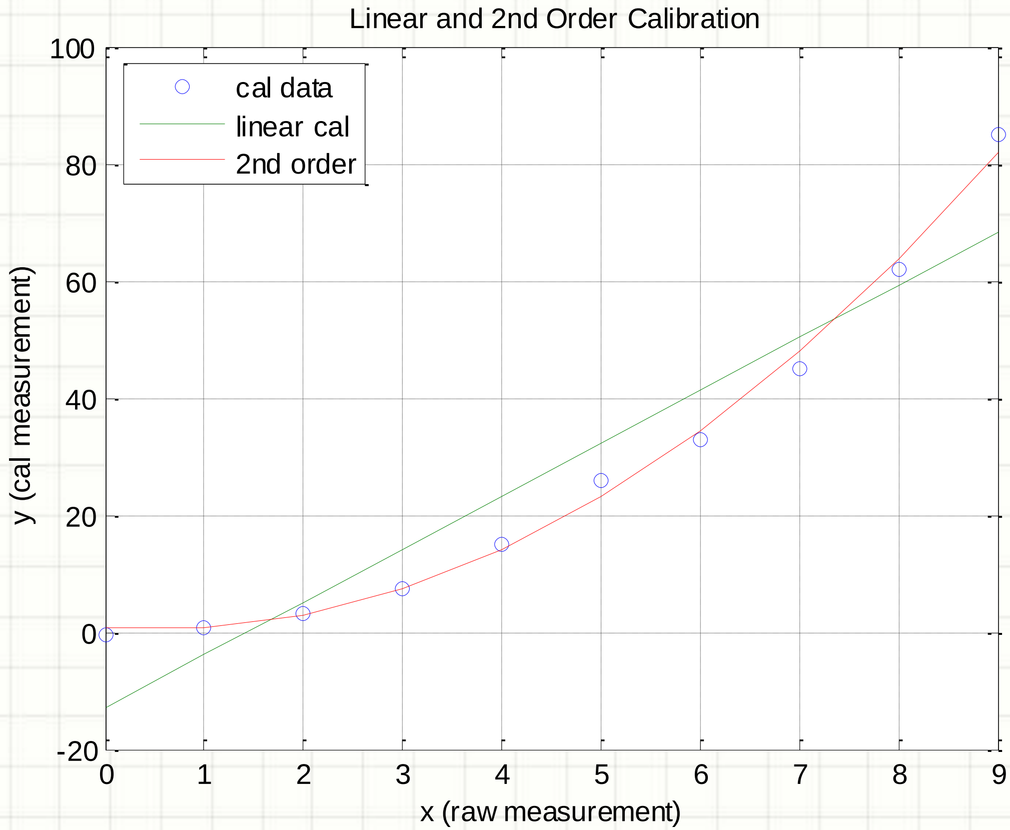
- The results are: p=[1.1362 -1.1825 0.6705], which means:

$$M_{cal} = 0.6705 - 1.1825 \times M_{raw} + 1.1362 \times M_{raw}^{2}$$



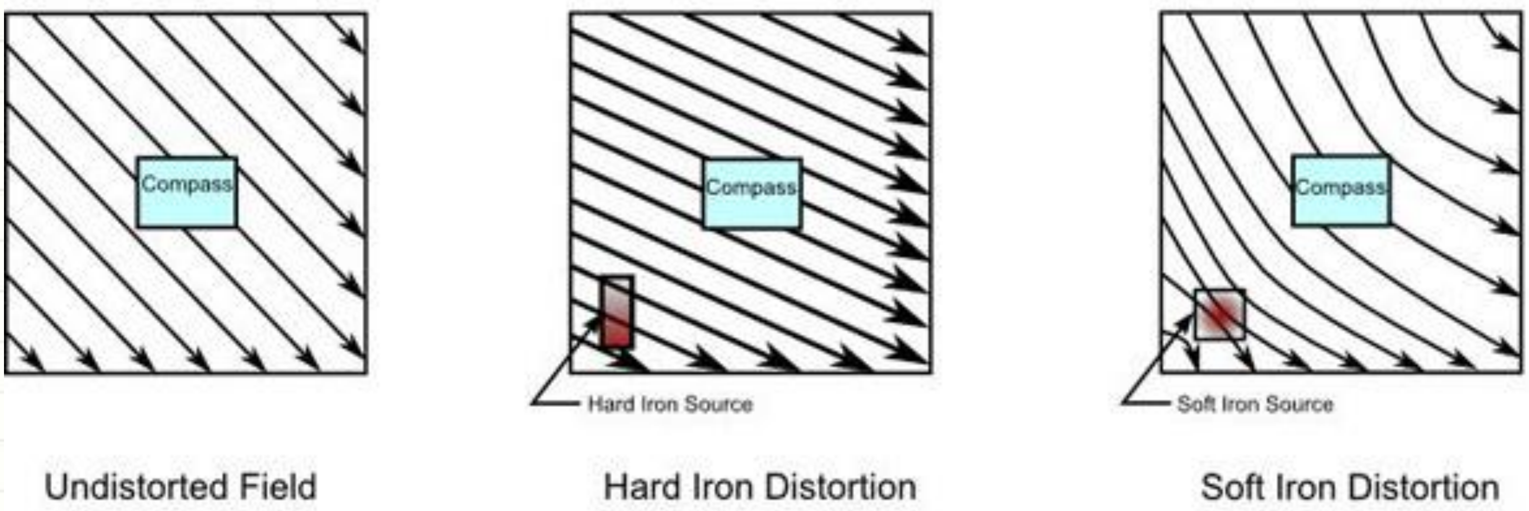
Second Order Calibration Result

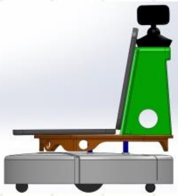
- The second order calibration is clearly shown better performance than the linear calibration here, but with added compexity;
- Whether it worth the effort to do higher-order calibration depend on the particular application.



Magnetometer Errors

- Magnetometers are commonly used for North finding (e-compass);
- The calibration of the magnetometer is however a highly complex process;
- The magnetometers readings are subject to two types of local distortions: hard iron and soft iron effects;
- The hard iron effect is caused by the presence of local permanently magnetized ferromagnetic components.





The Soft Iron Effect

- The soft iron effect is the interfering magnetic field induced by the local presence of normally unmagnetized ferromagnetic components;
- These components distorts the local magnetic field differently as it rotates, which can cause difficulty in magnetometer calibration.

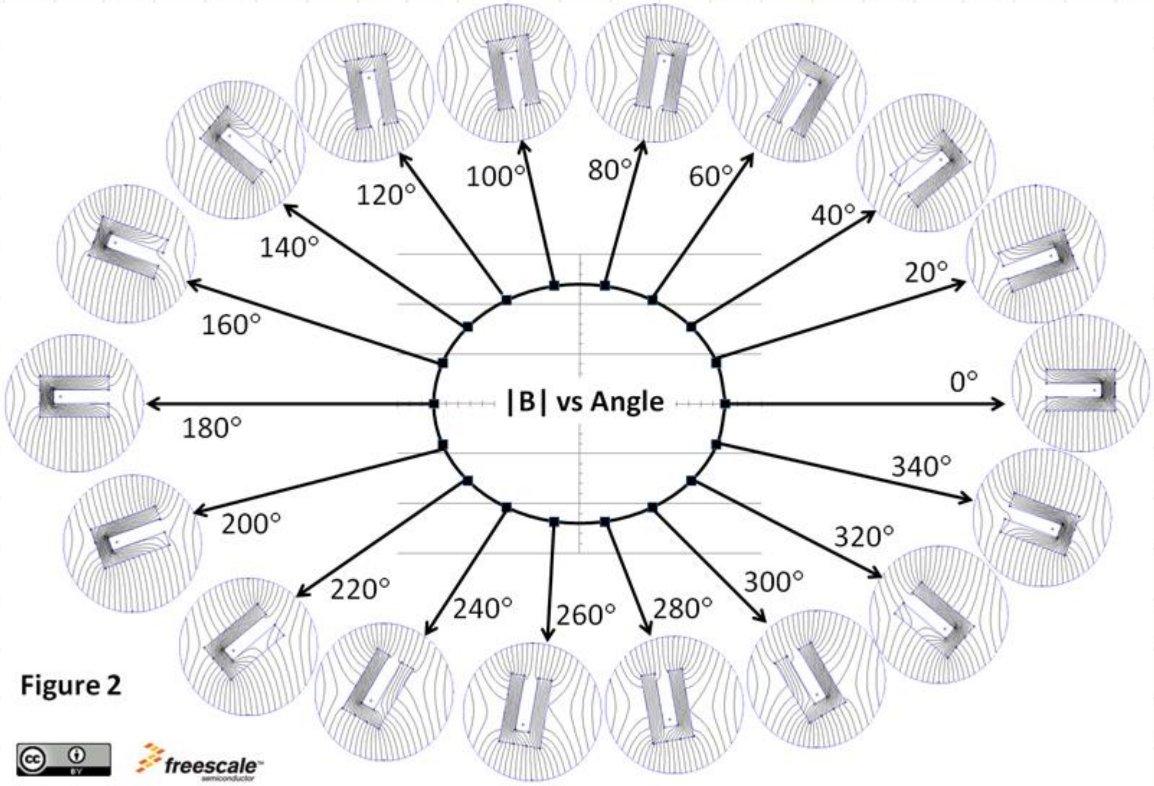
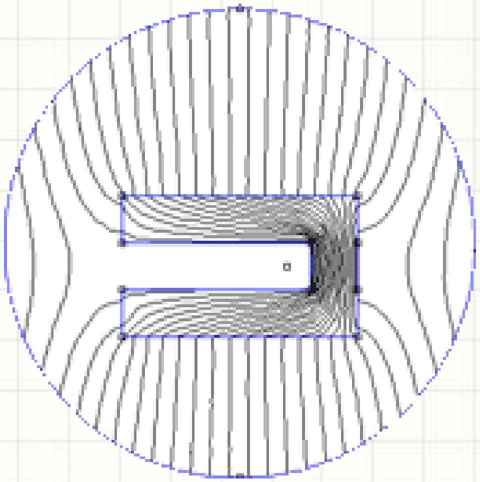
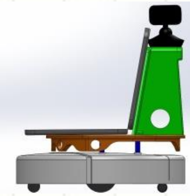


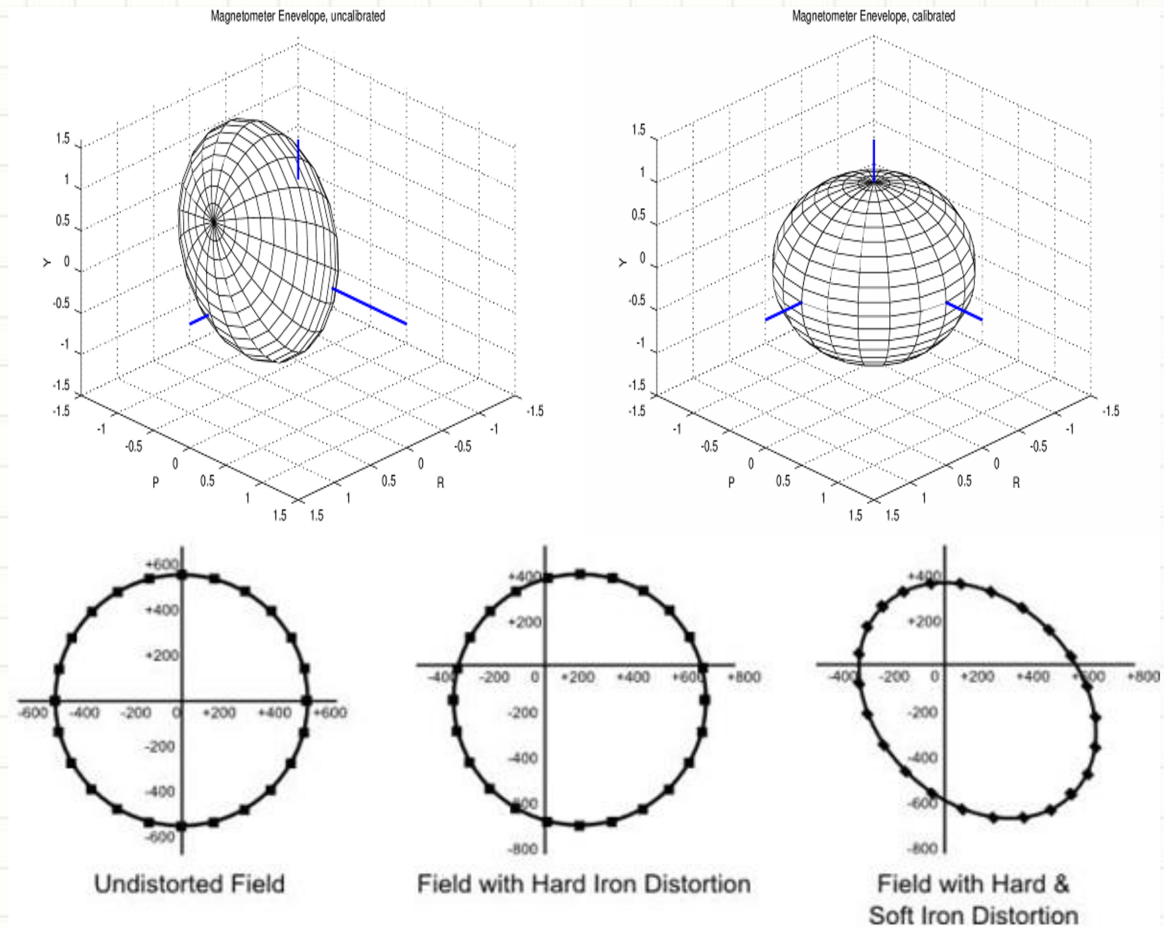
Figure 2





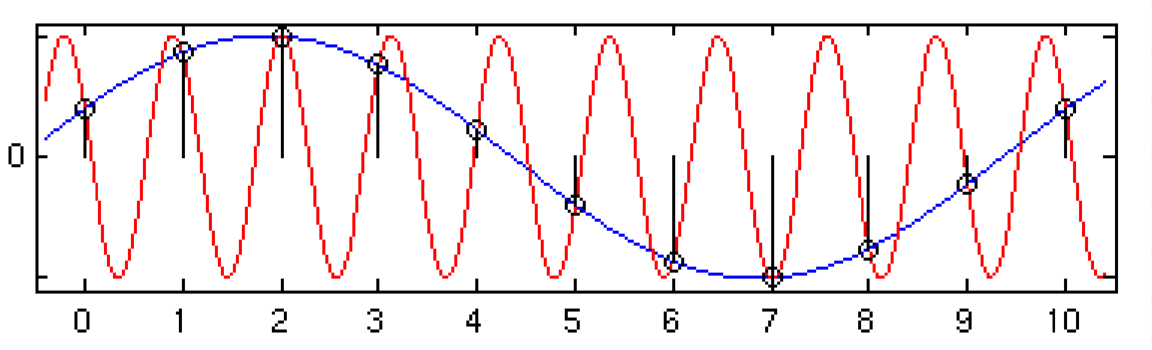
Magnetometer Calibration

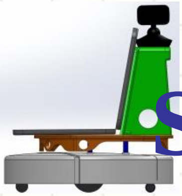
- There are a total of 10 parameters that needs to be calibrated for a set of 3-axis magnetometers;
- This include a geomagnetic field strength parameter, 3 offset parameters (hard iron), 3 scaling parameters (soft iron) and 3 rotation parameters (soft iron);
- We will learn more about the detailed calibration procedure later in this class.



Aliasing and Anti-Aliasing

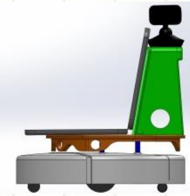
- Both spatial and temporal resolution of the sampling process are very important.
- If we sample below the a certain rate (undersampling), reconstructed signal will be different from the original. This phenomenon is called aliasing.
- Aliasing effect can often be reduced with an increased sampling rate or through the use of a low-pass filter.
- The cut-off frequency of the anti-aliasing filter should be below half of the sampling rate.





Shannon-Nyquist's Sampling Theorem

- If a band-limited signal x(t) contains no frequencies higher than B hertz, it must be sampled properly at a rate $f_s > 2B$.
- A signal sampled at $f_s = 2B$ is said to be Nyquist sampled. No information is lost if a signal is sampled at this rate, and no additional information is gained by sampling faster than this rate (*in theory*...).
- If the highest frequency B in the original signal is known, 2B is the lower bound on the sampling frequency for which perfect reconstruction can be assured. This is called the Nyquist rate.
- If instead f_s is known, $f_s/2$ gives an upper bound for frequency components of the signal to allow for perfect reconstruction. This upper bound $f_n = f_s/2$ is the Nyquist frequency.



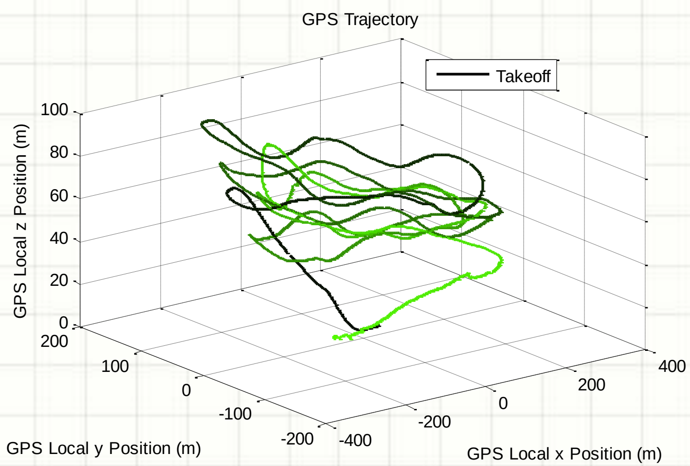
Practical Sampling

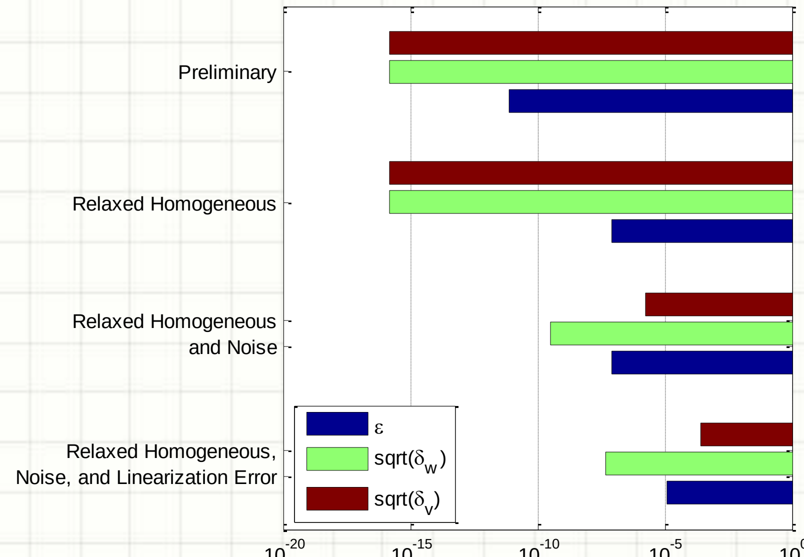
- Practical sampling rate range: $6 \le \dfrac{f_s}{f_{signal}} \le 40$
- This is called oversampling, which is the process of sampling a signal with a frequency significantly higher than the Nyquist rate;
- Oversampling can generally improve the response speed and also provide improved smoothness in the response;
- Higher sampling rate can also reduce the delay between a command change and the system response to the command change. In a digital control system, the command can be delayed up to a full sample period;
- The sampling rate has a major impact on how well a digital controller performs in the presence of plant disturbances.

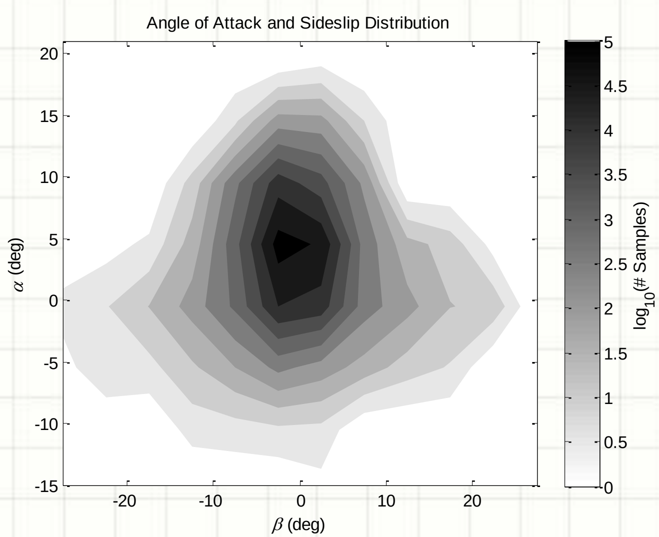
However, how fast can you sample is often limited by the hardware and cost. For Example, on SMART, we can only sample at ~10Hz because of the slow MATLAB computing.

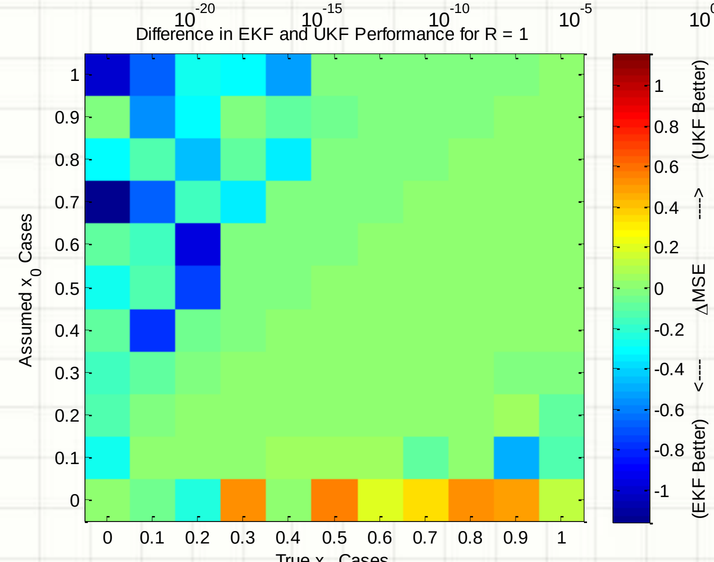
Representation of Engineering Data

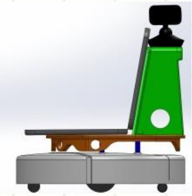
- MATLAB provides many ways to plot the data and you just have to be creative in using them.





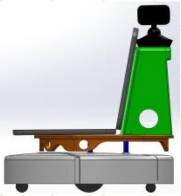






Summary

- Measurement error can arise anywhere in the measurement link;
- Systematic errors can be reduced through careful experiment design and calibration;
- Random errors can be reduced through statistical methods;
- Linear calibration is the simplest and most popular method;
- Magnetometers are affected by soft iron and hard iron distortions, thus difficult (but not impossible) to calibrate.
- A practical sampling rate is typically 3-20 times the Nyquist rate (if only considering the highest frequency component in the signal);



Further Reading

- Sensor Performance Specification: http://ieeexplore.ieee.org/iel5/37/20344/00939939.pdf?tp=&isnumber=&arnumber=939939
- MATLAB Polyfit: http://www.mathworks.com/help/matlab/data_analysis/programmatic-fitting.html
- Hard and soft iron magnetic compensation explained http://memsblog.wordpress.com/2011/03/22/hard-and-soft-iron-magnetic-compensation-explained/
- Presenting Engineering Data in MATLAB http://www2.statler.wvu.edu/~irl/IRL_WVU_Online_MATLAB_Plot_Tips_V1.0_06_28_2013.pdf